Let $\vec{b} = 4\vec{i} - \vec{j} - 8\vec{k}$ and $\vec{c} = -\vec{j} - 2\vec{k}$. **SCORE: _____ / 55 PTS**
Let L be the point $(-3, -5, 2)$.

[a] Write $3\vec{c} - 2\vec{b}$ in component form.

$$3\langle 0,-1,-2\rangle - 2\langle 4,-1,-8\rangle = \langle 0,-3,-6\rangle - \langle 8,-2,-16\rangle$$
$$= \langle -8,-1,10\rangle$$

[b] Find a vector of magnitude 2 in the same direction as $\vec{b}$.

$$2\left(\frac{1}{\|\vec{b}\|}\right)\vec{b} = 2\left(\frac{1}{\sqrt{81}}\right)\langle 4,-1,-8\rangle = \frac{2}{9}\langle 4,-1,-8\rangle$$
$$= \left\langle \frac{8}{9}, -\frac{2}{9}, \frac{-16}{9}\right\rangle$$

[c] Find a unit vector perpendicular to both $\vec{b}$ and $\vec{c}$.

$$\vec{b}\times\vec{c} = \langle 4,-1,-8\rangle \times \langle 0,-1,-2\rangle$$
$$= \langle 2-8, 0-8, -4-0\rangle$$
$$= \langle -6, 8, -4\rangle$$
$$\frac{1}{\|\vec{b}\times\vec{c}\|}(\vec{b}\times\vec{c}) = \frac{1}{\sqrt{116}}\langle -6,8,-4\rangle = \frac{1}{2\sqrt{29}}\langle -6,8,-4\rangle$$
$$= \left\langle \frac{-3}{\sqrt{29}}, \frac{4}{\sqrt{29}}, \frac{-2}{\sqrt{29}}\right\rangle$$

[d] Find the equation of the plane parallel to both $\vec{b}$ and $\vec{c}$, and passing through point L.
Write your final answer in general form $Ax + By + Cz + D = 0$.

$$-6(x+3)+8(y+5)-4(z-2)=0$$
OR $3(x+3)-4(y+5)+2(z-2)=0$
$$3x-4y+2z-15=0$$

[e] Let ℓ_1 be the line with parametric equation $x = 6-4t, \quad y = t+9, \quad z = 2t-7$.
Find the symmetric equation of the line parallel to ℓ_1 and passing through L.

$$\frac{x+3}{-4} = \frac{y+5}{1} = \frac{z-2}{2}$$

OR $-\frac{x+3}{4} = y+5 = \frac{z-2}{2}$

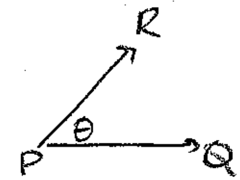
Let P be the point $(-3, -1, 5)$. Let Q be the point $(-7, 1, -1)$. **SCORE: _____ / 65 PTS**

Let R be the point such that $\overrightarrow{PR} = \langle 3, 2, 1 \rangle$.

[a] Find the co-ordinates of R.

$$(-3+3, -1+2, 5+1) = (0, 1, 6)$$

[b] In the triangle ΔPQR, find the measure of angle $\angle RPQ$.



$$\overrightarrow{PQ} = \langle -7 - -3, 1 - -1, -1-5 \rangle = \langle -4, 2, -6 \rangle$$

$$\cos^{-1} \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\|\overrightarrow{PQ}\| \|\overrightarrow{PR}\|} = \cos^{-1} \frac{-12+4-6}{\sqrt{56}\sqrt{14}} = \cos^{-1} \frac{-14}{2(14)} = \cos^{-1} -\frac{1}{2}$$

$$= \frac{2\pi}{3}$$

[c] Write $\overrightarrow{PR}$ as the sum of two orthogonal vectors, one of which is the projection of $\overrightarrow{PR}$ onto $\overrightarrow{PQ}$.

$$\frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\overrightarrow{PQ} \cdot \overrightarrow{PQ}} \overrightarrow{PQ} = \frac{-14}{56} \langle -4, 2, -6 \rangle = -\frac{1}{4} \langle -4, 2, -6 \rangle = \langle 1, -\tfrac{1}{2}, \tfrac{3}{2} \rangle$$

$$\langle 3, 2, 1 \rangle = \langle 1, -\tfrac{1}{2}, \tfrac{3}{2} \rangle + \langle 2, \tfrac{5}{2}, -\tfrac{1}{2} \rangle$$

[d] Find the area of the triangle ΔPQR.

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \langle -4, 2, -6 \rangle \times \langle 3, 2, 1 \rangle$$

$$= \langle 2 - -12, -(-4 - -18), -8-6 \rangle$$

$$= \langle 14, -14, -14 \rangle$$

$$\tfrac{1}{2} \|\overrightarrow{PQ} \times \overrightarrow{PR}\| = \tfrac{1}{2}(14) \|\langle 1, -1, -1 \rangle\| = 7\sqrt{3}$$

[e] Find the equation of the plane passing through P, Q and R. **Write your final answer in general form** $Ax + By + Cz + D = 0$.

$$14(x+3) - 14(y+1) - 14(z-5) = 0$$

OR $(x+3) - (y+1) - (z-5) = 0$

$$x - y - z + 7 = 0$$

If $< 6, a, -12 >$ is parallel to $< b, -2, 9 >$, find the values of a and b. **SCORE: _____ / 10 PTS**

$\langle 6, a, -12 \rangle = k\langle b, -2, 9 \rangle$

$6 = kb$

$a = -2k$

$-12 = 9k \longrightarrow k = -\frac{4}{3}$

$a = -2(-\frac{4}{3}) = \frac{8}{3}$

$6 = -\frac{4}{3}b \longrightarrow b = -\frac{9}{2}$

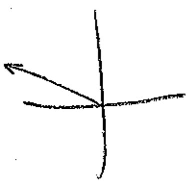
Let $\vec{r}$ be the vector with magnitude 4 and direction angle $\frac{\pi}{6}$. Let $\vec{s} = < -4\sqrt{3}, 4 >$. **SCORE: _____ / 20 PTS**

[a] Find the component form of $\vec{r}$.

$$\langle 4\cos\tfrac{\pi}{6}, 4\sin\tfrac{\pi}{6} \rangle = \langle 2\sqrt{3}, 2 \rangle$$

[b] Find the direction angle of $\vec{s}$.

$$\tan^{-1}\frac{4}{-4\sqrt{3}} = \tan^{-1} -\frac{\sqrt{3}}{3} = -\frac{\pi}{6} \text{ IN } Q_4$$



$$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

[c] If $\vec{r}$ represents a force, and $\vec{s}$ is the movement of an object that the force is applied to, find the work done.

$$\langle 2\sqrt{3}, 2 \rangle \cdot \langle -4\sqrt{3}, 4 \rangle = -8(3) + 8 = -16$$